Engineering Notes

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Gravity Gradient Stability of Satellites with Guy-Wire Constrained Appendages

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Introduction

SUFFICIENT conditions for the Liapunov stability of an Earth pointing, gravity gradient stabilized, rigid satellite are well known. Also, the effects of flexibility have been studied by numerous authors for a wide variety of configurations. In an attempt to better understand the fundamental physics of the effects of flexibility on the attitude stability of satellites, guy-wire constraints can be used to isolate and study the effects of various modes of appendage vibration on attitude stability. As a typical example of guy-wire constrained appendages, this Note investigates the effect of torsional vibrations of elastically supported solar panels on the attitude stability of Earth pointing, gravity gradient stabilized satellites. By using Liapunov's direct method, a set of sufficient conditions for stability are obtained involving the moments of inertia of the satellite, the orbital speed, and the torsional stiffness of the supporting shafts.

Problem Formulation

The satellite consists of a central body, of mass M (with principal body axes i, j, k), and two rectangular panels, each of mass m (with principal body axes i_1, j_1, k_1 and i_2, j_2, k_2), supported by massless, elastic shafts. The panels are assumed identical with width a, length b, and depth c, where $a > c$ and $b > c$. The shafts lie along the j axis of the central body and the panels are aligned so that the j_1 and j_2 axes lie along the j axis. The generalized coordinates used to describe the plane of each panel relative to the j, k plane are α_1 and α_2. The panels are attached so that $\alpha_1 = \alpha_2 = \pi/2$ when the panels are not vibrating. In this configuration, the i, j, k axes are principal axes for the entire satellite, provided no vibrations occur. A guy-wire system constrains the vibrations to a purely torsional mode (see Fig. 1 for schematic diagram of satellite).

The center of mass of the satellite is assumed to move in a circular orbit about a spherically symmetric planet. An equilibrium position of the satellite is defined to be one in which it is at rest with respect to a set of orbital axes a_1, a_2, a_3, which are defined as follows: 1) the origin is at the center of mass of the satellite, 2) a_1 is in the radial direction, 3) a_2 is tangent to the orbit in the direction of motion, and 4) a_3 is normal to the orbit plane. The axes i, j, k of the satellite are oriented relative to the orbital axes by a 2-1-3 rotation sequence through θ_2, θ_1, and ψ, respectively (see Ref. 1, p. 468). The $\theta_1 = \theta_2 = \psi = 0$ equilibrium position is the one studied in this paper.

Method of Analysis

There are many excellent references on the application of Liapunov's direct method. The approach adopted here is basically that found in Ref. 1. To apply Liapunov's direct method to our system, we must first construct a Liapunov function to test for stability. The kinetic energy T can be written as $T = T_0 + T_1 + T_2$, where T_2 is a homogeneous quadratic function in the generalized velocities, T_1 is a linear homogeneous function in the generalized velocities, and T_0 is a non-negative function of the generalized coordinates and time. By calculating the kinetic energy and the potential energy V, it can be shown that the system is non-natural and that the Lagrangian $L = T - V$ does not depend explicitly on time. This implies that the Hamiltonian $H = T_2 - T_0 + V$ is constant, i.e., $dH/dt = 0$. Thus, the Hamiltonian is a possible Liapunov function. To use the Hamiltonian as a testing function, it is necessary to determine under which conditions it is positive definite. Since T_2 is always positive definite, we only need to study the positive definiteness of $U = V - T_0$. This is accomplished by constructing the Hessian matrix for U and using Sylvester's criterion for determining positive definiteness.

Assuming that the largest satellite dimension is small compared to the orbital radius, the total kinetic energy of the satellite is

$$\begin{aligned} T = \tfrac{1}{2}\{ &(M + 2m)\Omega^2 R^2 + (A_c\omega_x^2 + B_c\omega_y^2 + C_c\omega_z^2) \\ &+ [(\omega_x \cos\alpha_1 - \omega_z \sin\alpha_1)^2 + (\omega_x \cos\alpha_2 - \omega_z \sin\alpha_2)^2]\, A_p \\ &+ [(\omega_y + \dot\alpha_1)^2 + (\omega_y + \dot\alpha_2)^2]\, B_p \\ &+ [(\omega_x \sin\alpha_1 + \omega_z \cos\alpha_1)^2 + (\omega_x \sin\alpha_2 + \omega_z \cos\alpha_2)^2]\, C_p \\ &+ 2m\ell^2(\omega_x^2 + \omega_z^2)\} \end{aligned} \tag{1}$$

where

$$\omega_x = \Omega\ell_{13} + \dot\theta_2 \cos\theta_1 \sin\psi + \dot\theta_1 \cos\psi$$

$$\omega_y = \Omega\ell_{23} + \dot\theta_2 \cos\theta_1 \cos\psi - \dot\theta_1 \sin\psi$$

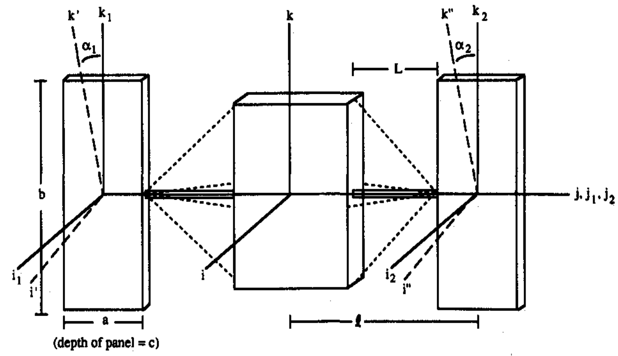

Fig. 1 Schematic diagram of satellite with guy wires attached.

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$\omega_z = \Omega\ell_{33} - \dot{\theta}_2 \sin\theta_1 + \dot{\psi}$

$\ell_{13} = -\sin\theta_2 \cos\psi + \cos\theta_2 \sin\theta_1 \sin\psi$

$\ell_{23} = \sin\theta_2 \sin\psi + \cos\theta_2 \sin\theta_1 \cos\psi$

$\ell_{33} = \cos\theta_2 \cos\theta_1$

A_p, B_p, C_p = principal moments of inertia of each panel about its principal body axes

A_c, B_c, C_c = principal moments of inertia of the central body about its principal body axes

Ω = orbital angular velocity

The potential energy consists of three parts: the potential energy due to the gravity gradient torques on the entire satellite, and the strain energies of the two panel-shaft systems. Thus, the expression for potential energy is

$$V = (-\Omega^2/4)[(3\ell_{11}^2 - 1)(I_{yy} + I_{zz} - I_{xx}) + (3\ell_{21}^2 - 1)(I_{xx} + I_{zz} - I_{yy}) + (3\ell_{31}^2 - 1)(I_{xx} + I_{yy} - I_{zz}) + 12(\ell_{11}\ell_{21}I_{xy} + \ell_{11}\ell_{31}I_{xz} + \ell_{21}\ell_{31}I_{yz})] + \frac{JG(\alpha_1 - \pi/2)^2}{2L} + \frac{JG(\alpha_2 - \pi/2)^2}{2L} \tag{2}$$

where J is the polar moment of inertia of each supporting shaft, G is the modulus of rigidity of each supporting shaft, and $I_{xx}, I_{yy}, I_{zz}, I_{xy}, I_{xz}$, and I_{yz} are the instantaneous moments of inertia of the entire satellite about the i, j, k axes and are given by

$$I_{xx} = A_c + 2m\ell^2 + A_p(\cos^2\alpha_1 + \cos^2\alpha_2) + C_p(\sin^2\alpha_1 + \sin^2\alpha_2)$$

$$I_{yy} = B_c + 2B_p$$

$$I_{zz} = C_c + 2m\ell^2 + A_p(\sin^2\alpha_1 + \sin^2\alpha_2) + C_p(\cos^2\alpha_1 + \cos^2\alpha_2)$$

$$I_{xy} = I_{yz} = 0$$

$$I_{xz} = (A_p - C_p)(\sin\alpha_1 \cos\alpha_1 + \sin\alpha_2 \cos\alpha_2)$$

In terms of θ_2, θ_1, and ψ,

$$\ell_{11} = \cos\theta_2 \cos\psi + \sin\theta_2 \sin\theta_1 \sin\psi$$

$$\ell_{21} = -\cos\theta_2 \sin\psi + \sin\theta_2 \sin\theta_1 \cos\psi$$

$$\ell_{31} = \sin\theta_2 \cos\theta_1$$

The function $U = V - T_0$ can now be constructed from Eqs. (1) and (2).

Stability Analysis

The Hessian matrix is a 5×5 matrix consisting of all of the second-order partial derivatives of U with respect to the generalized coordinates (i.e., $H^E = [\partial^2 U/\partial q_n, \partial q_m]$, where $q_1 = \theta_1$, $q_2 = \theta_2$, $q_3 = \psi$, $q_4 = \alpha_1$, and $q_5 = \alpha_2$). For the values $\theta_1 = \theta_2 = \psi = 0, \alpha_1 = \alpha_2 = \pi/2$, which represent the equilibrium point to be studied, the matrix has the form:

$$H^E = \begin{bmatrix} h_{11} & 0 & 0 & 0 & 0 \\ 0 & h_{22} & 0 & h_{24} & h_{25} \\ 0 & 0 & h_{33} & 0 & 0 \\ 0 & h_{42} & 0 & h_{44} & 0 \\ 0 & h_{52} & 0 & 0 & h_{55} \end{bmatrix}$$

where

$$h_{11} = \Omega^2[(C_c - B_c) + 2(A_p - B_p) + 2m\ell^2]$$

$$h_{22} = 4\Omega^2[(C_c - A_c) + 2(A_p - C_p)]$$

$$h_{24} = h_{25} = h_{42} = h_{52} = 4\Omega^2(A_p - C_p)$$

$$h_{33} = 3\Omega^2[(B_c - A_c) + 2(B_p - C_p) - 2m\ell^2]$$

$$h_{44} = h_{55} = 4\Omega^2(A_p - C_p) + JG/L$$

Sylvester's criterion may be used to obtain the following set of sufficient conditions for stability.

1) $h_{11} > 0$

2) $h_{22} > 0$

3) $h_{33} > 0$

4) $h_{22}h_{44} - h_{24}^2 > 0$

5) $h_{44}(h_{22}h_{44} - 2h_{24}^2) > 0$

(The computer program MACSYMA was used to perform the preceding calculations.)

Discussion of Results and Conclusions

The first result to note is that conditions 1 and 3 together imply condition 2. Furthermore, under the assumption that $b > c$, we have that $A_p - C_p > 0$, so $h_{44} > 0$, and condition 5 becomes

5′) $h_{22}h_{44} - 2h_{24}^2 > 0$

Therefore, we have that condition 5′ implies condition 4. Finally, we obtain a set of three sufficient conditions for stability, namely,

I) $(C_c - B_c) + 2(A_p - B_p) + 2m\ell^2 > 0$

II) $(B_c - A_c) + 2(B_p - C_p) - 2m\ell^2 > 0$

III) $[(C_c - A_c) + 2(A_p - C_p)][4\Omega^2(A_p - C_p) + JG/L] - 8\Omega^2(A_p - C_p)^2 > 0$

By defining $\omega_n = (JG/LB_p)^{1/2}$, $\beta = (A_p - C_p)/B_p$, and $Q = (A_p - C_p)/(A_c - C_c)$, we can write condition III as

III′) $\left(\frac{\omega_n^2}{\Omega^2}\right) > \frac{4\beta}{(2Q - 1)}$

where ω_n is the natural frequency of vibration for each panel shaft system.

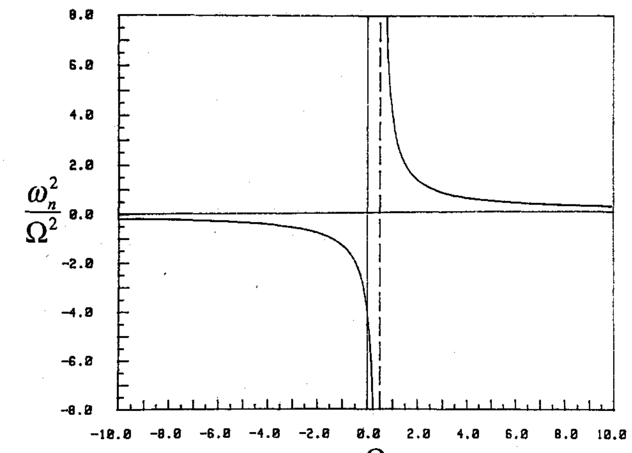

Fig. 2 Full-field Liapunov stability boundary for $b \gg c$ (i.e., thin panels for which $\beta \approx 1$).

Conditions I and II do not depend on the flexibility of the satellite. Since the moments of inertia A, B, and C of the entire undeformed satellite are defined as

$$A = A_c + 2m\ell^2 + 2C_p, \qquad B = B_c + 2B_p$$

$$C = C_c + 2m\ell^2 + 2A_p$$

we see that conditions I and II reduce to the requirement that $C > B > A$, the stability conditions for a gravity gradient stabilized rigid body discussed by DeBra and Delp.[2]

Condition III involves the flexibility of the satellite and III′ expresses this condition in terms of nondimensional parameters. Note that $\beta = (b^2 - c^2)/(b^2 + c^2)$. Therefore, since $b > c$, we have in general that $0 < \beta < 1$. Also, for most panels $b \gg c$, so $\beta \approx 1$. Assuming that $\beta = 1$, it is useful to look at a plot of the curve $(\omega_n^2/\Omega^2) = 4/(2Q - 1)$ since it gives the stability boundary of condition III′ (Fig. 2). Since $A_p - C_p > 0$, it can be shown that conditions I and II imply $Q > ½$ or $Q < 0$ so the domain of Q is $(-\infty, 0) \cup (½, \infty)$. Note that for $Q < 0$, the quantity $4\beta/(2Q - 1)$ is negative in which case condition III′ is trivially satisfied.

As the plot in Fig. 2 shows, the critical parameter value is $Q = ½$ since near this point the torsional stiffness becomes a significant factor. It is interesting to note that $Q = ½$ when $A = C$. Since $C > B > A$, we have that $Q \approx ½$ implies $A \approx B \approx C$. Thus, the region for which $Q \approx ½$ is near the origin of the k_1—k_2 DeBra-Delp graph.[2] Recall that the gravity gradient torque goes to zero at the origin of this graph. What these results indicate is that the torsional stiffness plays an increasingly significant role as the magnitude of the gravity gradient torque vector approaches zero.

The fact that condition III′ is easily satisfied as long as Q is not close to ½, along with the fact that condition III′ is automatically satisfied for all values of torsional stiffness if $Q < 0$, suggests that the use of guy wires to restrict vibrations to purely torsional modes can significantly improve attitude stability.

Acknowledgment

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References

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Results in Identification of a Flexible Structure Using Lattice Filters

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Introduction

THE main challenge in applying adaptive identification and control techniques to flexible structures arises from model order selection. Since the effective order of such systems varies with the number of substantially excited modes, the order of the algorithm cannot be fixed a priori. In the case of large space structures, maneuvers such as docking or on-orbit assembly may excite different sets of structural modes causing the system to be time varying as well as order varying. Knowledge of the system order (or maximum order) is typically needed for most identification and control methods. The lattice filter, recursive in time and order, does not require the model's order to be fixed a priori. This order-recursive property can be used to identify the system order on-line and to update the order of the parameter estimator for identification or control purposes.

The vector-channel lattice filters used here are based on the derivation in Ref. 1, where simulation results were presented for a hub-flexible beam. Also, in Ref. 2, vector-channel lattice filters were used to identify the natural frequencies of the NASA-Langley grid structure. The system in Ref. 2 was a high-order single-input/single-output (SISO) system with only slight variations in parameters (see Ref. 2 for further details). A somewhat similar approach was taken in Refs. 5 and 6 for the identification of flexible systems where order-recursive methods were used to obtain some of the results. Here, the lattice filter is solely used to identify an experimental truss structure (Fig. 1), which is a flexible multi-input/multi-output (MIMO) system. Previous experimentation on the truss[3] has shown its fundamental mode of vibration at 0.68 Hz and its 20th mode at 45 Hz. Preliminary results with comparisons to other identification methods were presented in Ref. 4. Here, the on-line identification of the structure's low-frequency dynamics using vector-channel lattice filters is considered for two important cases: 1) time-varying, in which the system undergoes a sudden variation in characteristics with a corresponding change in effective order; and 2) multi-input, in which the structure is excited simultaneously with all of its actuators (a total of four inputs). The vector-channel lattice filter is shown to perform well in both of these instances.

Problem Statement

The damped structural system's forced response is modeled by a finite-order differential equation of the following form:

$$M\ddot{x} + D\dot{x} + Kx = Bu \tag{1}$$

where $\boldsymbol{x}$ is the $n \times 1$ generalized displacement vector, and $\boldsymbol{u}$ is

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